Optimal harmonic confinement of a Brownian particle

Carlos A. Plata[1]

[1]Universidad de Sevilla, Seville, Spain

Harmonicity is ubiquitous in physics, and statistical mechanics is no exception. Beyond the convenience that harmonic potentials enable analytical treatments and naturally arise as quadratic approximations around equilibrium points, many experimental setups—such as optical tweezers—are accurately described by harmonic potentials. These successful experimental platforms have played a central role to accelerate many celebrated advances over the past decades within the context of stochastic thermodynamics, as well as in the application of (optimal) control theory at the mesoscopic scale.

In this contribution, we review and unify a class of optimal control problems for finite-time transformations of a Brownian particle confined by a harmonic potential. Both overdamped and underdamped regimes are considered, along with different choices of cost functionals. The control parameters are the intensity of the confinement and the temperature of the thermal bath. The impact of constraints on the controls is analyzed using Pontryagin's maximum principle. These results provide a systematic framework for simple yet fundamental protocols, which serve as building blocks for the design of mesoscopic heat engines.