Relative cluster entropy and emergent risk diversity in financial volatility

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Understanding correlation, heterogeneity, and risk in financial systems remains a central challenge. Although price time series are commonly modeled as weakly correlated stochastic processes, empirical evidence shows that volatility exhibits long-range correlations, non-Gaussian statistics, and pronounced multiscale structure, motivating approaches that capture emergent properties from coarse-grained dynamics.

Here we introduce an information-theoretic framework based on Kullback–Leibler (KL) cluster entropy to quantify correlation and diversity in financial volatility time series. Grounded in statistical-physics concepts such as coarse graining and relative information, the method provides a scale-resolved characterization of stochastic dynamics.

Clusters are obtained by intersecting a time series with its moving average, generating cluster durations encoding persistence and memory effects. This non-parametric procedure is related to first-passage and crossing properties of correlated stochastic processes, including fractional Brownian motion (FBM) [1]. From these sequences, we estimate the empirical distribution of cluster durations and compare it with a reference distribution representing uncorrelated Brownian motion with Hurst exponent $H = 1/2$.

The central observable, KL cluster entropy, measures the relative entropy between empirical and reference cluster distributions. Unlike Shannon entropy, which quantifies intrinsic uncertainty within a single distribution [2], KL entropy captures deviations from a benchmark stochastic process [3]. For power-law cluster statistics associated with fractional processes, KL cluster entropy depends directly on the Hurst exponent, providing an inferential measure of long-range dependence.

Applying the framework to high-frequency realized volatility of five major equity indices (S&P500, NASDAQ, DJIA, DAX, and FTSEMIB), we find evidence of positive long-range correlations ($H > 1/2$). KL cluster entropy is maximal at short cluster durations and decays toward zero at longer scales, indicating convergence toward uncorrelated dynamics. This behavior complements Shannon cluster entropy, which increases with cluster duration and captures growing uncertainty at large timescales [4].

We further introduce a KL-based cluster diversity index, obtained by aggregating relative entropy across cluster scales. This index enables a consistent ranking of assets according to their contribution to systemic risk diversity. The resulting entropy-based portfolio weights are smoother and more stable than those derived from mean-variance or Sharpe-ratio optimization [5].

The KL cluster entropy framework reveals correlation strength and scale dependence in financial systems, placing markets within the class of complex adaptive systems characterized by multiscale organization and long-range correlations [6].

References:

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