Power laws, symmetries and conservation in nonequilibrium mass transport processes

Punyabrata Pradhan[1], Animesh Hazra[1], Tanmoy Chakraborty[1], Anirban Mukherjee[2]

[1]S.N. Bose National Centre For Basic Sciences, Kolkata, India, [2]Institute of Physics, Academia Sinica, Taipei 11529, Taiwan

We investigate a broad class of conserved-mass transport processes on a d-dimensional hypercubic lattice ($d > 1$) with multidirectional hopping dynamics that preserve all lattice symmetries. We demonstrate that these systems generically exhibit long-ranged, power-law correlations in the steady state, even far from a phase transition, if any. The mechanism underlying this behavior is the presence of multidirectional hopping events, in which multiple particles or chunks of masses simultaneously hop out from a site in different directions. Such processes violate detailed balance, making the system intrinsically out of equilibrium and not being described by the Boltzmann–Gibbs distribution, while respecting global conservation laws and lattice symmetries.

Employing a coarse-grained hydrodynamic description substantiated by exact microscopic analysis, we show that the steady-state density–density correlation function decays algebraically at large distances for all spatial dimension $d > 1$. The corresponding exponents are determined by spatial dimension, symmetries, and the conserved quantities. The strength of the power law is calculated exactly for several models and expressed in terms of the bulk-diffusion coefficient and the Onsager matrix (or, the mobility tensor). Essentially the multidirectional hopping of masses qualitatively modifies the noise statistics in the hydrodynamic evolution of density field. Strikingly, in models having axial bidirectional hopping of masses, the scaled density-density (two-point) static correlation functions (connected) have a generic asymptotic form. In particular, additional conservation laws beyond global density—most notably center-of-mass conservation—lead to enhanced suppression of long-wavelength fluctuations, leading to anomalously strong correlations.

To support our general arguments, we focus on arguably the simplest representative model systems. The systems considered are continuous-time Markov processes with nearest-neighbor hopping on a hypercubic lattice. They are closed (mass conserving), diffusive (reflection-symmetric hopping rates), spatially homogeneous (translation invariant), and isotropic (invariant under lattice rotations). Consequently, the steady state carries no net mass current. We show that multidirectional hopping dynamics induces a nonanalyticity in the static structure factor, leading to the emergence of power-law correlations.

Our framework provides a general and unified explanation for the emergence of long-ranged correlations in systems with center-of-mass–conserving dynamics, recently proposed as minimal models of disordered hyperuniform states. Furthermore, we show that unequal-time current–current correlations also exhibit algebraic decay. However, their temporal scaling reflects faster relaxation compared to ordinary diffusive systems governed by a single conserved density field. Together, these results identify multidirectional hopping as a generic route to scale invariance in mass-conserving systems.