

A new structure of informational functionals and inequalities

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In this talk we present recent results dealing with certain generalizations of informational inequalities of moment-entropy, Stam, and Cramér-Rao type applied to probability density functions. In particular, we show the existence of a mirrored range of the parameters involved in the Lutwak-Yang-Zhang inequalities. More precisely, we obtain new inequalities with the same formal formulation as the Lutwak-Yang-Zhang generalized inequalities of moment-entropy and Stam type, but with different minimizers. In fact, the minimizers of the new inequalities have a divergence behavior in the borders of their support, in a clear contrast with the bounded behavior of the minimizers in the "classical" range of the parameters. In addition, we unexpectedly find that, although in the classical case the Cramér-Rao inequality is obtained as the mere product of entropy-moment and the Stam inequality, in the mirrored domain there is no Cramér-Rao counterpart.

As the main tool in the proofs, we have employed a pair of mutually inverse transformations, called "up" and "down" transformations, acting on suitable classes of probability density functions. The key point is the fact that, the down transformation relates the family of p -th absolute moments with the family of Rényi entropy functionals, but at the same that, surprisingly, it also relates the family of Rényi entropy functionals with the family of two-parameter Fisher information measures defined by Lutwak-Yang-Zhang.

Moreover, we define new informational functionals inspired by "up" and "down" transformations. Furthermore, new sharp informational inequalities involving both new and classical informational functionals are proved and their minimizers are explicitly computed. Remarkably, some of these new minimizers correspond with the family of generalized Beta distributions. Finally, we show that the latter structure of informational inequalities allow to us to find new upper bound for the products of classical inequalities such as the Stam inequality. More precisely, we complete the bounds of the Stam inequality by adding a new upper bound depending on the regularity conditions of the probability density functions. These new upper bounds are also sharp, and the maximizers are explicitly computed.

References:

1) Iagar, R. G., & Puertas-Centeno, D. (2025). A new pair of transformations and applications to generalized informational inequalities and Hausdorff moment problem. *Communications in Nonlinear Science and Numerical Simulation*, 151, 109091.

2) Iagar, R. G., & Puertas-Centeno, D. (2025). Through and beyond moments, entropies and Fisher information measures: new informational functionals and inequalities. *Physica D: Nonlinear Phenomena*, 134928.