Violations of the Thermodynamic Uncertainty Relation in Autonomous Underdamped Heat Engines

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Thermodynamic uncertainty relations (TURs) bound the precision of thermodynamic currents in nonequilibrium steady states and, for autonomous heat engines, imply trade-offs between fluctuations, dissipation, power, and efficiency. Their validity in underdamped autonomous devices, however, remains far from fully understood. This question is especially relevant for microscopic thermal machines, where inertia and internal resonances can strongly affect current statistics while preserving autonomous steady operation.

In this contribution, we discuss recent results showing that underdamped autonomous stationary heat engines can robustly violate the long-time TUR and the associated power-efficiency-constancy trade-off. We first consider a nonlinear setup in which two underdamped continuous degrees of freedom are coupled to a two-level system and jointly operate as an autonomous heat engine. In this regime, resonant internal dynamics allow one degree of freedom to act as an effective internal drive for the other, leading to strong suppression of current fluctuations without external periodic forcing. Remarkably, the regime of maximal suppression can be inferred from mean-current measurements alone, providing an experimentally accessible signature of enhanced constancy and TUR violation.

We then outline a minimal extension based on a discrete ratchet coupled to an underdamped harmonic oscillator. In a suitable time-scale separation regime, this model becomes analytically tractable and yields a closed expression for the TUR ratio, making the fluctuation-suppression mechanism especially transparent. Taken together, these results show that inertia can qualitatively reshape precision-dissipation trade-offs in autonomous thermal machines and suggest design principles for precise microscopic engines and autonomous clocks operating far from equilibrium, beyond standard overdamped bounds.

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