Martingale forecasts of heat dissipation in non-equilibrium relaxations

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We develop a theoretical ('martingale') framework to describe the statistical properties of the heat associated with non-stationary relaxations of hetereogeneous diffusion processes [1]. Using our framework, we obtain analytical formulae that enable forecasting the heat dissipated by a spherical microscopic particle subject to gravity and interacting hydro-dynamically with a rigid wall [2]. In particular, our theory reveals that the average heat dissipated as a function of time may be conserved, decrease, or increase as a function of time till relaxation to equilibrium depending on the initial position of the particle. We illustrate these results with realistic experimental parameters and rationalize them in terms of the pioneering martingale formalism of stochastic thermodynamics [3] which we extend to relaxations in arbitrary potentials and arbitrary space-dependent friction profiles.

References:
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[2] Giovanni Volpe et al, PRL 104, 170602 (2010)
[3] Edgar Roldan et al, Adv. Phys. 72 (1-2), 1 (2023).