

Following the yellow brick road: hidden correlations along random geodesics

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First-passage percolation (FPP) characterizes geodesics between two fixed points at a distance d on graphs whose edges are endowed with random link-times [1]. Indeed, geodesics in square lattices under a weak disorder eventually fall into the Kardar-Parisi-Zhang (KPZ) universality class. Indeed, fluctuations of arrival times grow as d^β , while their lateral deviations grow as d^ζ , with $\beta = 1/3$ and $\zeta = 2/3$, and the arrival times themselves follow a Tracy-Widom distribution [2].

Picture yourself traveling along such a geodesic. The link-times along the edges that you visit must take lower values than the rest, because they have been specifically chosen in order to yield a minimal arrival time. We may ask, how much lower? We have addressed this question in a recent work [3], characterizing the statistical properties of the "selected link-times" (SLT). Our analysis reveals universal power-law decays with the end-to-end distance for both the average and standard deviation of the SLTs.

Even though the original link-times are uncorrelated, can the SLTs present correlations among them? Statisticians are very aware of the so-called collider bias, or Berkson's paradox: selecting individuals above a certain threshold in the sum of different uncorrelated traits induces correlation between them. This bias has surprising effects in genetics, risk assessment or astrophysics. Indeed, in our case, SLT along a geodesic present long-range correlations characterized by a critical exponent within the KPZ class [3].

Finally, the SLT distribution must violate the conditions of the central limit theorem because, otherwise, arrival times would be Gaussian, instead of following a Tracy-Widom distribution. Indeed, we have found an intricate set of high-order correlations, with a strong resemblance to the predictions of a conformal field theory (CFT).

References:

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