Characterizing the dynamics of driven Brownian particles in optical lattices

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Transport processes in biological and soft-matter systems, such as diffusion of proteins in periodic cellular structures, or particle transport through porous media, can often be highly complex. Brownian motion in a periodic force field serves as a minimal model of equilibrium and nonequilibrium transport, and understanding the essential mechanisms underlying such systems. Earlier work revealed phenomena such as giant diffusion near a critical tilt of the potential [1], and characterized the long-time transport properties such as the drift velocity and the effective diffusion coefficient [2 ,3]. However, the complete characterization of the particle dynamics remains largely unexplored.

Here, we solve the Fokker-Planck equation for a single overdamped particle in a (tilted) cosine potential and derive the intermediate scattering function (ISF), which characterizes the full spatio-temporal dynamics. Importantly, this quantity is directly accessible in experiments through dynamic light scattering, particle tracking, or dynamic differential microscopy. Beyond the standard ISF, we introduce a generalized ISF probing correlations between two wave vectors, which are non-zero when the wave vectors differ by multiples of the reciprocal lattice, reflecting the spatial periodicity of the potential. We analyze the full-time dependence by computing observables including the variance, diffusivity, skewness, and the non-Gaussian parameter.

Exploiting spatial periodicity via Bloch's theorem, we formulate the dynamics in a spectral framework, providing analytic expressions for the ISF and the propagator in terms of numerically computed eigenvalues and eigenfunctions. Closed expressions for low-order moments and cumulants are obtained using a Taylor expansion of the ISF for small wave numbers. Results are validated via Brownian dynamics simulations and compared to the harmonic approximation and deterministic limit. In the case without tilt, the system reduces to an equilibrium process, and we compare our analytical results to experiments on colloidal particles in optical lattices [4]. In particular, the ISF provides distinctive features that can be exploited for experimental calibration and assess their consistency.

We identify the ratio of tilt to potential amplitude as the key control parameter governing the dynamics, which shows trapping-dominated and drift–diffusive regimes. For small tilts, the ISF exhibits a pronounced two-step relaxation: an initial plateau resulting from trapping, followed by a decay due to barrier hopping. This regime is further characterized by peaks in the skewness and non-Gaussianity of the displacement distribution. For strong tilts, finite drift leads to pronounced oscillations in both the real and imaginary parts of the ISF, as well as in the low-order moments.

References:

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