Run-and-tumble dynamics with nonreciprocal transitions among three velocity states

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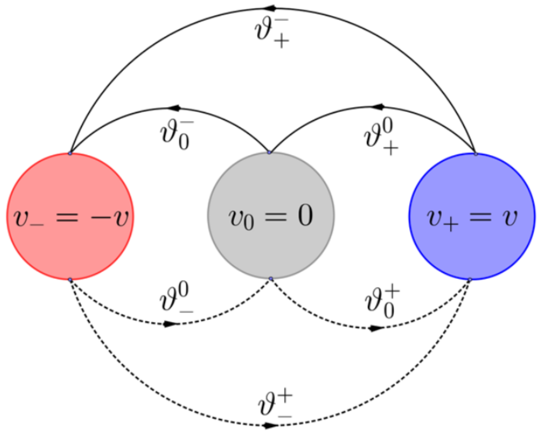
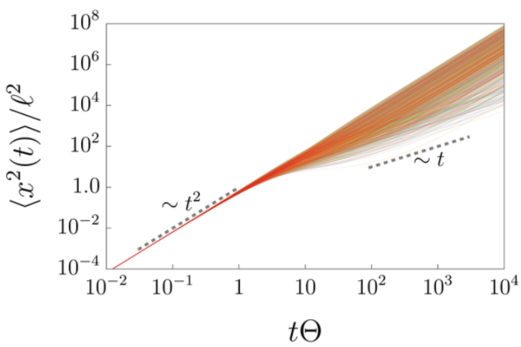
Active motion has proven a fertile ground for exploring nonequilibrium physics, where energy consumption at the microscopic scale generates emergent transport phenomena at larger scales. A paradigmatic model that describe active motion is the run-and-tumble dynamics, widely used to describe in simplistic way bacterial motility. Standard formulations typically assume reciprocal transitions between self-propelling velocity states, thereby preserving microscopic reversibility. However, many realistic systems operate under inherently asymmetric conditions. In this work, we investigate how breaking this reciprocity at the level of internal dynamics reshapes macroscopic transport.





We present a one-dimensional run-and-tumble particle whose self-propulsion velocity switches among three states, $\{-v, 0, +v\}$, with nonreciprocal transition rates. This minimal extension explicitly violates detailed balance and introduces a controlled mechanism of internal irreversibility. Using a combination of analytical methods and kinetic Monte Carlo simulations, we systematically explore the resulting transport properties across the space of transition rates.

We find that nonreciprocity generates a broad spectrum of transport regimes. Depending on the asymmetry of the transition network, the system exhibits long-time ballistic transport, enhanced diffusion, or standard diffusive behavior. Exact expressions are obtained for the drift velocity, effective diffusion coefficient, and moments of the position distribution, revealing how statistical properties are governed by the stationary distribution of internal states.

A key result is the identification of a manifold in parameter space where the effective drift vanishes, leading to purely diffusive behavior despite the absence of microscopic reversibility. This highlights a nontrivial route by which equilibrium-like transport emerges from intrinsically irreversible dynamics. Away from this manifold, imbalances between forward and backward transitions induce persistent currents and ballistic motion, with the drift velocity directly controlled by the degree of nonreciprocity.

We further characterize the transient regime and show that the inclusion of a resting state qualitatively alters short-time dynamics compared to the standard two-state model. In particular, the propagation of probability distributions departs from simple wave-like behavior and exhibits non-Gaussian features, which are captured through the intermediate scattering function and the time evolution of the kurtosis.

Our results establish a clear link between internal-state irreversibility and emergent transport properties in active systems. The proposed framework provides a versatile platform to investigate nonequilibrium dynamics beyond reciprocal models, with potential applications ranging from biological motility to the design of synthetic active matter with tunable transport characteristics.

Reference:

J. C. R. Romo-Cruz and F. J. Sevilla, Run-and-tumble dynamics with nonreciprocal transitions among three velocity states, Physical Review E 112, 054115 (2025).