A Benchmark for Self-Organized Criticality

Alexander Shapoval[1]

[1]University of Lodz, Lodz, Poland

Self-organized criticality (SOC) — the property of complex systems to evolve toward a critical state without parameter tuning — captures what has been described as "how nature works". SOC emerges from the separation of timescales: slow stress accumulation and its rapid release. By revisiting the Bak–Tang–Wiesenfeld sandpile on the $L \times L$ lattice, the first model of SOC, we numerically explore the bulk of the size–frequency relationship (which widens as L^2 and precedes the rapid decay) and find two distinct regimes that persist when L grows toward infinity. The transition point between the regimes exhibits non-trivial scaling, L^a, with $1 < a < 2$, with respect to the system size. Intermediate-scale events form a power-law region, which occupies almost the entire bulk when small lattices are used. The second regime, which could be mistakenly confused with a bump when the lattice size is up to $L = 1024$, is identified. Its functional description is still unclear, due to its limited extent even for the largest available system. One may assume that this large-scale regime is also described by a power-law. Under this hypothesis, we propose a method to estimate its exponent. We found that if two power-law regimes indeed exist, their exponents are similar, though noticeably distinct. The L^a scaling of the transition point implies that the intermediate-scale power-law regime, which prevails in available systems, occupies only a tiny part of the size–frequency relationship on the double logarithmic scale as the system size approaches infinity. Accordingly, the large-scale regime occupies more of the size–frequency relationship, though it widens more slowly than L^2.

Despite this limitation, we demonstrate that the BTW mechanism does capture the essence of SOC. By introducing a mix of slow and fast time scales to the original model, we obtain a modified system whose size–frequency relationship follows a true power law, with a rapid cutoff at extreme avalanches. In general, the BTW mechanism, as defined by Bak, Tang, and Wiesenfeld, introduces a system that resides in the (small) neighborhood of one exhibiting a second-order phase transition, understood in terms of self-invariance. To reach the point representing the actual phase transition, one should adjust the mechanism, as we are proposing in this paper.