

Unifying Kappa Distribution Models for Non-Equilibrium Space Plasmas: A Superstatistical Approach Based on Moments

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We present a unified statistical framework to describe non-thermal particle distributions in space plasmas based on superstatistics and minimal physical assumptions about particle–environment interactions. Observations across the heliosphere consistently reveal velocity distribution functions (VDFs) that deviate from Maxwellian equilibrium and exhibit suprathermal tails, commonly modeled using kappa distributions. However, the physical interpretation of their parameters, particularly temperature, remains debated.

In this work, we show that kappa-like distributions naturally emerge from a superstatistical formulation when the most probable kinetic energy of a particle is assumed to depend linearly on the energy of its surrounding environment. This minimal closure leads to a family of distributions that reproduce both classical kappa parameterizations as particular cases. Crucially, we demonstrate that the resulting single-particle distribution can be fully characterized by the statistical properties of an underlying inverse-temperature distribution $P(\beta)$, specifically its mean β_s and relative variance u .

Within this framework, we provide a consistent interpretation of temperature beyond equilibrium. We show that different definitions of temperature used in the literature correspond to distinct statistical estimators of $P(\beta)$, such as its mean or mode. In particular, we demonstrate that the temperature associated with the second moment of the velocity distribution corresponds to a specific statistical property of $P(\beta)$, rather than a unique thermodynamic quantity. This clarifies longstanding ambiguities in the interpretation of kappa distributions and explains the coexistence of different parameterizations.

Furthermore, we propose a moment-based formulation of the particle distribution that avoids the need to explicitly define temperature, showing that the full distribution can be reconstructed from a small number of moments (e.g., second and fourth in velocity space). This provides a compact and physically transparent description of non-equilibrium plasmas and establishes a direct link between observable quantities and the underlying statistical structure.

Our results offer a unified perspective on kappa distributions as manifestations of temperature fluctuations and provide a robust theoretical framework to interpret spacecraft observations and kinetic simulations of space plasmas.