Hydrodynamics of ultra-confined systems

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We investigate the hydrodynamic description of a dilute gas composed of hard spheres confined between two parallel, infinite plates. The confinement is characterized as ultra-confinement because the separation between the plates is of the same order as the particle diameter, which restricts the motion of the particles to being quasi-bidimensional.

The foundation of this description is a kinetic theory based on a height-averaged distribution function. A Boltzmann-like equation for this function was previously derived through a formal BBGKY hierarchy, justifying its validity to first order in the density. This kinetic equation accounts for the specific geometry of the system by integrating over the allowed collision angles permitted by the narrow gap between the plates. It was also demonstrated that this description satisfies a version of the H-theorem and leads to a well-defined global equilibrium state for the confined fluid [1].

Using this kinetic equation as a starting point, we perform a formal Chapman-Enskog expansion to derive the governing hydrodynamic equations. In the initial approach, we describe the state of the fluid through four hydrodynamic fields: the two-dimensional number density, the parallel components of the flow velocity, and a single three-dimensional temperature. This derivation provides explicit analytical expressions for transport coefficients, including the thermal conductivity, dynamic shear viscosity, and bulk viscosity. Each of these coefficients depends on the plate separation distance. Notably, the bulk viscosity, which has a purely geometric origin in this system, exhibits a divergence as the gap width approaches the particle diameter. This divergence leads to non-physical predictions for the speed and absorption of sound in the limit of extreme confinement.

To address this issue, we propose an extended hydrodynamic description. In the regime of extreme confinement, the collision geometry only allows for nearly frontal impacts between the spheres. This restriction causes the kinetic energy associated with motion parallel to the plates to be approximately conserved during collisions. Consequently, the local isotropy of the temperature is broken, necessitating the use of separate temperature fields for the parallel and vertical directions. This leads to a five-field Chapman-Enskog expansion that eliminates the divergences and captures the transition of the transport coefficients from 3d to 2d values from first principles, in very good agreement with event-driven molecular dynamics simulations.