Group entropies, information theory and complexity

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Generalized entropies play a central role in characterizing the complexity of a wide range of deterministic and stochastic processes. In the first part of this talk, we introduce the concept of group entropy, a class of information measures endowed with an intrinsic group-theoretical structure that emerges from the requirement of composability under the union of statistically independent systems.

We generalize the Shannon–Khinchin axioms by replacing additivity with composability. Using formal group theory from algebraic topology, this yields an infinite family of entropy measures, the group entropies, which are generally non-trace-form. The classical Rényi entropy is the first example in this hierarchy.

We show that complex systems can be classified into universality classes, each characterized by a phase-space growth rate and described by an associated group entropy that quantifies complexity.

As a first application, we extend the Bandt–Pompe permutation entropy method for time series analysis. A key challenge is the super-exponential growth of admissible permutations with sequence length, in the presence of randomness or observational noise. We adopt a classification based on the growth rate of admissible permutations, independent of whether data are deterministic or stochastic. This leads to three main classes: exponential, sub-factorial, and factorial growth.

The group-theoretical permutation entropies extend the standard definition, providing new tools for complexity analysis. They offer a unified framework for ordinal analysis from deterministic dynamics to white noise.

The second part of the talk develops a link between group entropies and machine learning, leading to a class of Mirror Descent (MD) optimization algorithms. By employing group-theoretical mirror maps defined through generalized logarithms and their inverses (group exponentials), we obtain adaptive update rules suited to different data geometries and statistical structures.

We introduce mirror duality, which enables interchange between these functions and their inverses under suitable learning-rate conditions. By tuning parameters of the generalized logarithms, the method adapts to data statistics while maintaining convergence properties. This framework enhances flexibility and suggests new approaches to regularization and natural gradient methods in machine learning and deep learning. The proposed algorithms are tested on large-scale simplex-constrained quadratic programming problems.

This work is based on joint work in collaboration with J. M. Amigó (CIO-Elche), A. Cichocki (Polish Academy of Science, Warsaw), H. Jensen (Imperial College, London), P. Jizba (Czech Technical University, Prague).

Physical applications of the theory of Group Entropies will be presented in the related talks by H. J. Jensen (Group entropies and statistical physic) and P. Jizba (Group entropies and black-holes thermodynamics).

Short Bibliography:

1. H. J. Jensen, P. Jizba and P. Tempesta, arxiv:2603.02385 (2026)
2. A. Cichocki and P. Tempesta, arxiv:2603.08651 (2026)
3. H. J. Jensen, P. Tempesta, Entropy 26, 266 (2024).
4. J.M. Amigó, R. Dale and P. Tempesta, Chaos, 32, 112101 (2022) (featured article)
5. P. Tempesta, H. J. Jensen, Nature - Scientific Reports, 10, 1-11 (2020).