

# Generalized Lotka-Volterra model on sparse asymmetric graphs: local equations and Random Matrix Theory

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Real ecosystems, characterised by sparse and asymmetric interactions, represent highly complex disordered systems that pose a fundamental challenge for theoretical analysis. To address this, we analyse the generalised Lotka-Volterra model on sparse graphs, with and without demographic noise [1]. Drawing on the success of local closures in spin-glass dynamics [2] and combinatorial optimisation, we adapt this framework to Fokker-Planck equations to characterise stationary states in sparse networks. By deriving local Fokker-Planck equations and employing a mean-field closure, we obtain an efficient characterisation of stationary states for both symmetric and asymmetric interaction structures. Validated against direct numerical simulations, this method recovers recent results [3] and dense-network limits. Furthermore, it enables the systematic mapping of the phase diagram for sparse asymmetric networks for the first time. Specifically, our results not only show that sparsity shifts the phase boundaries from those predicted in the dense limit but also reveal the existence of a phase characterised by multiple stable fixed points. This stands in stark contrast to dense asymmetric systems, where the single-equilibrium phase gives way to a regime in which all fixed points are unstable, leading to the emergence of chaotic dynamics. We match these newly mapped phase transitions with insights from Random Matrix Theory [4], demonstrating a direct link between the spectral properties of sparse random matrices and the dynamical behaviour of these sparse ecological models. Ultimately, our local equation framework provides a versatile, computationally efficient tool for exploring complex dynamics in realistic ecological communities, with broad applications to a wide class of complex systems in statistical physics.

## References:

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