

Geometric complexity in thermodynamics: A complexity unattainability principle

Tan Van Vu¹, Keiji Saito²

¹Yukawa Institute for Theoretical Physics, Kyoto University, Japan, ²Department of Physics, Kyoto University, Japan

The quest to control and manipulate physical systems at the microscopic level is fundamentally constrained by the laws of thermodynamics. A cornerstone of these limitations is the third law, which asserts the unattainability of absolute zero—and, by extension, perfect purity—in a finite number of operational steps. In the context of modern information processing, such as resetting a qubit or erasing a classical memory bit, this principle implies the impossibility of achieving a perfect target state without infinite physical resources. While this unattainability principle has been rigorously quantified for specific state-to-state transitions, establishing a universal, dynamics-independent bound for implementing arbitrary, state-agnostic reset maps has remained a prominent open challenge.

Perfect state-reset operations are critical for both thermodynamic control and quantum computation, particularly for memory initialization and quantum error correction. Yet, physical maps are invariably subject to execution errors due to finite operational time, limited energetic budgets, or bounded control bandwidths. Existing frameworks often treat these resources separately, making it difficult to formulate a unified law governing the ultimate cost of map implementation across different physical platforms. In this work, we resolve this by introducing a framework that captures these disparate physical costs through a singular, unifying measure: geometric complexity. By examining continuous paths of physical maps evolving on geometric manifolds, we analyze the fundamental limits of general physical map implementation. Our geometric approach is highly general, encompassing both classical stochastic maps and quantum channels to provide a versatile geometric perspective on dynamical evolution.

Our central result is a rigorous trade-off relation between the execution error of a physical map and its geometric complexity. We prove that the geometric complexity of any physical map fundamentally lower-bounds its execution error. Consequently, we demonstrate that attempting to achieve a perfect, zero-error state-reset operation definitely forces the geometric complexity to diverge. Because our measure of geometric complexity naturally scales with physical constraints—incorporating operational time, thermokinetic cost, and control bandwidth—this divergence implies that a perfect reset demands infinite time, unbounded energy, or infinite bandwidth. We formalize this generalized limitation as the “complexity unattainability principle.” This principle holds universally across both classical and quantum regimes, establishing a strict, unified geometric limit on the physical realization of reset operations.