Symmetric Teleparallel Gravity and Information Geometry

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Information geometry (IG) [1] is a useful framework for studying certain probability distribution families. IG incorporates non-Riemannian geometric structures into spaces of statistical models and probability distributions within the field of information science.

In our previous work, we studied gradient-flow in IG from various perspectives and in relation to different fields of physics, such as analytical mechanics, geometric optics and thermodynamics, as well as Weyl's gauge symmetry [2]. Through these studies, we have shown that IG is connected to other fields that were previously thought to be unrelated.

Recently, non-metricity formulations of gravity, particularly Symmetric Teleparallel Gravity (STG) or Symmetric Teleparallel Equivalent of General Relativity (STEGR)[3], have attracted attention. General relativity is well known to be based on curvature, whereas STG is based on non-metricity.

Traditionally, IG has been considered to be unrelated to modified gravity theories. However, both theories have one thing in common: they are both based on non-Riemannian geometry. Especially in STG, curvature and torsion are vanishing, and the connection is independent, meaning the covariant derivative of the metric does not necessarily vanishes. Because of this independence, one can choose a specific coordinate system (gauge) known as the "coincident gauge," in which the affine connection vanishes globally.

An affine connection is generally characterized by curvature, torsion and non-metricity. Since curvature and torsion are both zero in dually flat IG spaces, non-metricity must play a role.

In this contribution, we will demonstrate some of the connections between IG and STG by considering the gradient flow in IG as the trajectories of a point particle.

The fundamental geometric structures in IG are studied from the perspective of non-metricity [4]. The non-metricity tensor in IG is characterized by the Amari–Centsov tensor. Starting from this, we have derived an explicit expression for the α-connection.

References:

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