Dirac-Equation Signal Processing

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Topological signals are variables or features associated with both nodes and edges of a network. Recently, in the context of topological machine learning, great attention has been devoted to signal processing of such topological signals. Most previous topological signal processing algorithms treat node and edge signals separately and work under the hypothesis that the true signal is smooth or well approximated by a harmonic eigenvector of the higher-order Laplacian, which may be violated in practice. Here, we propose Dirac-equation signal processing (DESP), a framework for efficiently reconstructing true signals on nodes and edges jointly and relaxing the assumption that signals are (almost) harmonic.

The proposed physics-inspired algorithm is based on the spectral properties of the topological Dirac equation, which extends the Dirac equation of high-energy physics to arbitrary networks. The topological Dirac equation introduces a mass parameter that controls the relative scale between node and edge signal components of its eigenstates. DESP reconstructs topological signals by jointly learning the mass and energy parameters in an unsupervised way.The algorithm reduces to Laplacian signal processing (LSP) and to the previous proposed Dirac signal processing (DSP). In the general case, DESP can outperform both LSP and DSP, as it processes node and edge signals jointly, unlike LSP, and allows them to have different scales — a situation that commonly arises in real data but that DSP cannot accommodate.

We further propose the iterated Dirac-equation signal processing (IDESP) algorithm to reconstruct true signals that are not aligned to a single eigenstate of the topological Dirac equation. IDESP iterates the DESP procedure to decompose the signal into multiple eigenstates one at a time, which is necessary for processing real signals that are in general linear combinations of several eigenstates.

We validate DESP and IDESP on synthetic signals defined on network models as well as on real ocean drifter trajectory data from the Global Ocean Drifter Program. The results demonstrate improved performance of DESP over both LSP and DSP, and show that IDESP can further reduce reconstruction error on real data. This work opens the way for further use of the topological Dirac equation in topological machine learning.