Geometric Entropy: What Any Observer Can Know

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Information is never destroyed. In classical mechanics, this is Liouville's theorem: Hamiltonian flow is symplectic and phase-space volume is conserved. In quantum mechanics, it is unitarity: the von Neumann entropy is exactly constant under Schrodinger evolution. Both are theorems, not assumptions. What can happen — and does happen, generically — is that information becomes inaccessible: real, still encoded in the full theory, but unreachable by any physically admissible operation on any observer's side of a geometric boundary. Entropy is the measure of that inaccessibility. We demonstrate this through three results sharing a single mathematical structure.

In Hamiltonian systems with $H(t) = H(-t)$, a Krylov-complexity construction — Hermiticity of $[H, .]$ under the Hilbert-Schmidt inner product, the three-term Lanczos recurrence, and Favard's uniqueness theorem — proves $b_n^{fwd} = b_n^{bwd}$, giving $\lambda_L^{fwd} = \lambda_L^{bwd}$: the quantum KS entropy is identical in both time directions. The arrow of time is not in the dynamics. It enters through a separate geometric mechanism: chaotic contraction drives phase-space structure below the quantum resolution scale $l_\hbar$ at $t_c = \lambda^{-1}\ln(\delta_0/l_\hbar)$, beyond which the time-reversed microstate is unreachable by any physically admissible operation. The information is still there — unitarity guarantees it — it has become geometrically inaccessible. This predicts sigmoid fidelity decay in Loschmidt echo experiments, perturbation-independent and rate-set by intrinsic Lyapunov exponents, consistent with three decades of experiments.

A second result concerns black hole entropy. Four successive symplectic reductions trace the eight-dimensional cotangent bundle to a single observable degree of freedom per surface element: the null constraint and geodesic-flow quotient yield the space of null rays; the symplectic complement of the generator submanifold selects two canonical pairs; the relational-clock isotropic quotient removes one; causal halving removes half of the last. The accessible fraction is (1/2 pair)/(2 pairs) = 1/4. The Bekenstein-Hawking entropy $S = A/4l_P^2$ counts inaccessible information — degrees of freedom causally excluded from any exterior observer, not absent from the theory. No field equations invoked.

A third result concerns horizon thermality. The inaffinity eigenvalue equation $\xi^\nu \nabla_\nu \xi^\mu = \kappa \xi^\mu$ uniquely fixes $\tau = \kappa^{-1}\ln\lambda$, forcing branch-structured modes $u_\omega \sim \lambda^{-i\omega/\kappa}$. Positive-frequency admissibility excludes the u_ω^+ branch, fixing $|\beta_\omega|/|\alpha_\omega| = \exp(-\pi\omega/\kappa)$. The symplectic identity then gives $\langle N_\omega \rangle = 1/(\exp(2\pi\omega/\kappa) - 1)$ at $T = \kappa/2\pi$. The thermal spectrum is the imprint of causal inaccessibility on the exterior observer's incomplete basis. No field equations. No Killing field. No bifurcate horizon.

In each case, a geometric condition partitions phase space into what any observer can access and what they cannot. The inaccessible portion is not gone — Liouville's theorem and unitarity jointly guarantee it. Entropy is the measure of that irreducible inaccessibility. The second law does not reflect a breakdown of microscopic reversibility. It reflects the geometry of what any observer can reach.