Entropy production in non-Gaussian active matter: fluctuation theorem for Levy-Ito SDE and deep learning algorithm

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We present a general framework for deriving entropy production rates (EPRs) in active matter systems driven by non-Gaussian active fluctuations in the form of the Levy-Ito stochastic differential equation. The existing challenges include the lack of theoreticaly and numerical tools to analyze the path distribution for such non-Gaussian stochastic dynamics and most of existing results are restricted to OU process or special jump process.

Employing the probability-flow equivalence technique, we rigorously obtain an entropy production (EP) decomposition formula for this general non-Gaussian system by studying the log ratio of probabilities for the forward and reverse trajectories. We demonstrate that the EP, Δs_{tot}, satisfies a detailed fluctuation theorem, $\rho_{\mathcal{R}}(\Sigma)/\rho_{\mathcal{R}}(-\Sigma) = e^{\Sigma}$, which holds for the distribution $\rho_{\mathcal{R}}(\Sigma)$ defined as the probability of observing a value Σ of the quantity $\mathcal{R} \equiv \Delta s_{\text{tot}} - B_{\text{act}}$, where B_{act} is a path-dependent random variable associated with active fluctuations. Moreover, an integral fluctuation theorem, $\langle e^{-\mathcal{R}} \rangle = 1$, and the generalized second law of thermodynamics, $\langle \Delta s_{\text{tot}} \rangle \geq \langle B_{\text{act}} \rangle$, follow directly.

The statistical effect from non-Gaussian Levy noise is partically exhibits in B_{act} and related to the so-called Levy score function in the effective velocity. Our results hold under steady-state conditions and can be straightforwardly extended to arbitrary initial states. In the limiting case where active fluctuations vanish, these theorems reduce to the established results of stochastic thermodynamics for Brownian Ito stochastic dynamics.

Building on this theoretical foundation, we introduce a deep-learning-based methodology for efficiently computing the EP, utilizing the Lévy score we proposed in our previous paper (accepted by SIAM Numerial Analysis). The method transports an ensemble of particles determistically in a time-stepping style by estimating the Levy score in a self-consistent way. To illustrate the validity of our approach, we apply it to two representative systems: a Brownian particle in a periodic active bath and an active polymer composed of an active Brownian cross-linker interacting with passive Brownian beads. Our work provides a unified framework for analyzing EP in active matter and offers practical computational tools for investigating complex nonequilibrium behavior.