Equilibrium distributions for strongly nonlinear many-body systems

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The equilibrium distribution function is essential for determining macroscopic observables in statistical physics, serving as the foundation for computing thermodynamic and correlation properties. Conventional methods, such as those based on the random phase approximation, are primarily designed for weakly nonlinear systems; in some cases, they can also be applied to strongly nonlinear systems that admit a transformation into near-integrable forms. However, these approaches often break down when nonlinear interactions are genuinely strong and cannot be perturbatively treated. To address this, we propose a theoretical framework based on the generalized energy equipartition principle, which enables the systematic derivation of equilibrium distributions and dispersion relations in strongly nonlinear many-body systems. Unlike standard methods, our framework intrinsically incorporates nonlinear effects and remains valid even when the spectral structure undergoes qualitative changes, such as mode merging or new resonance emergence. We validate the approach on three prototypical models: the nonlinear Schrödinger equation, the Majda–McLaughlin–Tabak model, and the Fermi–Pasta–Ulam–Tsingou-β model. In strongly nonlinear regimes, our method shows significant improvements over existing techniques. Extensive numerical simulations confirm the accuracy and robustness of the framework across diverse systems.