Data-Driven Ginzburg–Landau Dynamics for Non-Gaussian Spatial Fields

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Spatially extended systems with incomplete or irregular observations arise in a wide range of natural and engineered settings, including geophysical flows, materials microstructures, and environmental fields. A common challenge in these contexts is the reconstruction of spatial fields that exhibit both non-Gaussian statistics and nontrivial spatial correlations. Standard statistical approaches, such as Gaussian random fields or kriging [1], rely on restrictive distributional assumptions, while more flexible machine learning methods, such as support vector machine [2], often lack explicit representation of spatial interactions or physical interpretability.

In this work we propose a data-driven framework for spatial field reconstruction based on stochastic time-dependent Ginzburg–Landau (TDGL) dynamics [3]. The central idea is to infer an effective local potential directly from the empirical distribution of the observed data, thereby encoding non-Gaussian statistics at the level of a coarse-grained free-energy functional. Spatial correlations are introduced through a gradient term that penalizes local variations of the field, leading to a stochastic relaxation dynamics that evolves an initial configuration toward an equilibrium ensemble consistent with both the prescribed marginal distribution and spatial continuity.

In the special case of a quadratic potential, the model reduces to a Gaussian random field with a covariance structure induced by a local diffusion-like operator, establishing a direct connection to Matérn-type covariance models widely used in spatial statistics. The proposed approach can therefore be interpreted as a non-Gaussian extension of such models, in which the marginal distribution is no longer constrained to be Gaussian but is instead learned from data. This provides a natural bridge between stochastic partial differential equations, statistical field theory, and data-driven modeling.

The numerical implementation is based on a semi-implicit spectral scheme that enables efficient simulation of the TDGL dynamics on regular grids [4]. The stochastic formulation further allows generation of ensembles of equilibrium configurations, facilitating uncertainty quantification. Applications to synthetic and real datasets with strongly non-Gaussian characteristics demonstrate that the method reproduces reasonably well both empirical distributions and spatial correlations while remaining computationally efficient.

The results suggest that data-driven Ginzburg–Landau dynamics provide a flexible and physically interpretable framework for modeling complex spatial systems, opening new perspectives at the interface of statistical physics and data-driven inference.

References:

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